# Math 53 Discussion Problems Oct 10

1. Find $\frac{\partial w}{\partial v}$ when $u = 0, v = 0$, if $w = x^2 + \frac{y}{x}, x = u - 2v + 1, y = 2u + v - 2$

2. Find $\frac{\partial w}{\partial u}$ when $u = \frac{1}{2}, v = 1$, if $w = xy + yz + xz, x = u + v, y = u - v, z = uv$

3. Find $\frac{\partial w}{\partial r}$ when $r = 1, s = -1$, if $w = (x + y + z)^2, x = r - s, y = \cos(r + s), z = \sin(r + s)$

4. A function $f$ is called homogeneous of degree $n$ if it satisfies the equation $f(tx, ty) = t^n f(x, y)$ for all $t$, where $n$ is a positive integer.

   (a) Show that if $f$ is homogeneous of degree $n$,

   $$x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = nf(x, y)$$

   (b) Show that if $f$ is homogeneous of degree $n$,

   $$f_x(tx, ty) = t^{n-1} f_x(x, y)$$

5. Suppose that the equation $F(x, y, z) = 0$ implicitly defines each of the three variables $x, y$ and $z$ as functions of the other two. If $F$ is differentiable and $F_x, F_y$ and $F_z$ are all nonzero, show that

   $$\frac{\partial z}{\partial x}\frac{\partial x}{\partial y}\frac{\partial y}{\partial z} = -1$$